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离散互联模糊系统的分散控制及稳定性条件*

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摘　要: 本文针对一类离散互联模糊系统，提出了一种新的模糊分散控制器的设计方法，给出了控制系统稳定的充分条件，应用Lyapunov函数法和线性矩阵不等式理论证明了模糊分散控制系统的稳定性．所提出的模糊分散控制方法和稳定性条件克服了已有模糊分散控制算法的保守性．仿真结果进一步验证了所提的模糊分散控制理论的正确性和方法的有效性．

关键词: 互联模糊系统；模糊分散控制；充分条件；线性矩阵不等式

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1　引言

基于T-S模糊模型的非线性不确定系统制的研究已经取得了很大的进展，并获得了许多理论研究成果[1-8]，已成为处理非线性不确定复杂系统的控制问题的有效方法．特别是自Tanaka[2]给出系统稳定的充分条件之后，对于模糊系统的稳定性条件的研究受到众多学者的广泛关注．文献[9-11]相继给出了模糊系统更加放宽的稳定性条件，所设计的模糊控制有效地克服了系统稳定的保守性[11]．

众所周知，由于工业过程的日趋复杂化和大型化，互联系统或称大系统已成为控制界的研究热点之一．受T-S模糊控制系统研究的启发，近年来，国内外一些学者把T-S模糊系统控制方法引入非线性互联系统的控制中来，用T-S模糊模型对非线性互联系统进行描述或建模，在此基础上给出模糊分散控制器的设计和稳定性条件[12-17]．但是，现有的模糊分散控制设计方法都是基于文献[2]的稳定性条件设计的，所给出系统的稳定条件具有保守性．为了克服其稳定性条件的保守性，最近，文献[17]基于文献[10]的模糊系统稳定性条件，提出了基于T-S模糊模型的连续互联系统新的稳定条件，得到了很好的控制效果．然而，文献[17]的模糊分散控制方法和稳定性条件仅限于连续模糊互联系统，对于离散互联模糊系统来说，其稳定性条件都是在原有的研究成果[1-5]基础上给出，因此，还是相当保守的．

本文针对一类离散互联模糊系统，在已有研究成果的基础上，提出了一种模糊分散控制设计方法，并给出了使离散互联模糊系统稳定的新条件，应用Lyapunov函数和线性矩阵不等式理论证明了模糊分散系统的稳定性．因此本文所提出的模糊分散控制设计方法扩展了现有离散互联模糊系统的分散控制结果．

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2 模糊分散控制器的设计

考虑由N个T-S模糊子系统组成的一类离散互联模糊系统，其第i个模糊子系统由下面的$l\,(l=1,2,\cdots,r_i)$条模糊规则规定如下：

系统规则：如果$z_{i1}(t)$是F_{i1}^l，$z_{i2}(t)$是$F_{i2}^l,\cdots$，$z_{ig}(t)$是F_{ig}^l，则

$$x_i(t+1)=A_i^l x_i(t)+B_i^l u_i(t)+\sum_{j=1,i\neq j}^{N} R_{ij}^l x_j(t), \tag{1}$$

式中$F_{iq}^l\,(q=1,2,\cdots,g)$为模糊集合，$z_{iq}(t)=[z_{iq}(t),\cdots,z_{iq}(t)]^{\mathrm{T}}$是可测系统变量，即前件变量．$A_i^l$, B_i^l为系统矩阵和输入矩阵；R_{ij}^l为子系统S_i和S_j之间的关联项；$x_i(t)\in R^{n_i}$为第i个模糊子系统的状态向量；$u_i(t)\in R^{m_i}$为第i个模糊子系统的控制向量．

采用单点模糊化，乘积推理和平均加权去模糊化，则第i个模糊子系统的模型为

$$x_i(t+1)=\sum_{l=1}^{r_i}\mu_i^l\big(z_{iq}(t)\big)\Big[A_i^l x_i(t)+B_i^l u_i(t)+\sum_{j=1,i\neq j}^{N} R_{ij}^l x_j(t)\Big], \tag{2}$$

$\mu_i^l(z_{iq}(t))$为式(3)所定义的归一化隶属函数，具有下列基本特点

$$\alpha_i^l\big(z_{iq}(t)\big)=\prod_{q=1}^{g}F_{iq}^l\big(z_{iq}(t)\big);\quad \mu_i^l\big(z_{iq}(t)\big)=\frac{\alpha_i^l(z_{iq}(t))}{\sum_{l=1}^{r_i}\alpha_i^l(z_{iq}(t))};\quad \mu_i^l\big(z_{iq}(t)\big)\geq 0;\quad \sum_{l=1}^{r_i}\mu_i^l\big(z_{iq}(t)\big)=1, \tag{3}$$

$F_{iq}^l(z_{iq}(t))$为前件变量$z_{iq}(t)$在模糊集F_{iq}^l上的隶属度．

采用平行分布补偿(PDC)技术，设计第i个模糊子系统的控制器

$$u_i(t)=-\sum_{l=1}^{r_i}\mu_i^l\big(z_{iq}(t)\big)K_i^l x_i(t), \tag{4}$$

式中K_i^l为待确定的静态反馈增益矩阵，把(4)代入(2)得到模糊闭环系统为

$$\begin{aligned}x_i(t+1)\ =\ &\sum_{l=1}^{r_i}\sum_{l=1}^{r_i}\mu_i^l\big(z_{iq}(t)\big)\mu_i^m\big(z_{iq}(t)\big)\big[A_i^l-B_i^l K_i^m\big]x_i(t)\\&+\sum_{l=1}^{r_i}\sum_{j=1,i\neq j}^{N}\mu_i^l\big(z_{iq}(t)\big)R_{ij}^l x_j(t).\end{aligned} \tag{5}$$

控制目标：设计模糊分散控制器(4)，使得模糊分散控制系统在Lyapunov函数意义下渐近稳定．

3 主要结果

本节给出保证离散互联模糊系统(5)在Lyapunov函数意义下，渐近稳定的条件及控制设计算法．

定理 如果存在正定矩阵$Q_i\,(i=1,2,\cdots,N)$；矩阵N_i^l和$Y_i^{slm}\,(l,m,s=1,2,\cdots,r_i)$满足不等式(6)-(9)，其中，当$s=l\neq m$时，$Y_i^{llm^{\mathrm{T}}}=Y_i^{mll}$；当$l,m,s$两两不等时，$Y_i^{slm^{\mathrm{T}}}=$

Y_i^{mls}，$Y_i^{lsm^{\mathrm{T}}} = Y_i^{msl}$，$Y_i^{sml^{\mathrm{T}}} = Y_i^{lms}$，则有模糊状态分散控制器 (4) 使得离散互联模糊系统 (5) 是 Lyapunov 函数意义下渐近稳定的.

$$\begin{pmatrix} -Q_i & 0 & \Pi_1 \\ * & -Q_i & Q_i R_{i_\Sigma}^{l^{\mathrm{T}}} \\ * & * & -Q_i \end{pmatrix} < Y_i^{lll}, \quad l = 1, 2, \cdots, r_i, \tag{6}$$

$$\begin{pmatrix} -3Q_i & 0 & \Pi_2 \\ * & -3Q_i & Q_i(2R_{i_\Sigma}^{l^{\mathrm{T}}} + R_{i_\Sigma}^{m^{\mathrm{T}}}) \\ * & * & -3Q_i \end{pmatrix} < Y_i^{llm} + Y_i^{lml} + Y_i^{llm^{\mathrm{T}}}, \quad l, s = 1, 2, \cdots, r_i, \quad l \neq m, \tag{7}$$

$$\begin{pmatrix} -6Q_i & 0 & \Pi_3 \\ * & -6Q_i & 2Q_i(R_{i_\Sigma}^{l^{\mathrm{T}}} + R_{i_\Sigma}^{m^{\mathrm{T}}} + R_{i_\Sigma}^{s^{\mathrm{T}}}) \\ * & * & -6Q_i \end{pmatrix} < Y_i^{slm} + Y_i^{lsm} + Y_i^{sml} + Y_i^{slm^{\mathrm{T}}} + Y_i^{lsm^{\mathrm{T}}} + Y_i^{sml^{\mathrm{T}}},$$

$$s = 1, 2, \cdots, r_i - 2, \quad l = s+1, s+2, \cdots, r_i - 1, \quad m = l+1, l+2, \cdots, r_i, \tag{8}$$

$$\begin{pmatrix} Y_i^{1l1} & Y_i^{1l2} & \cdots & Y_i^{1lr_i} \\ Y_i^{2l1} & Y_i^{2l2} & \cdots & Y_i^{2lr_i} \\ \vdots & \vdots & \ddots & \vdots \\ Y_i^{r_il1} & Y_i^{r_il2} & \cdots & Y_i^{r_ilr_i} \end{pmatrix} < 0, \quad l = 1, 2, \cdots, r_i, \tag{9}$$

其中

$$\Pi_1 = Q_i A_i^{l^{\mathrm{T}}} - N_i^{l^{\mathrm{T}}} B_i^{l^{\mathrm{T}}}, \quad \Pi_2 = 2Q_i A_i^{l^{\mathrm{T}}} + Q_i A_i^{m^{\mathrm{T}}} - (N_i^{m^{\mathrm{T}}} + N_i^{l^{\mathrm{T}}}) B_i^{l^{\mathrm{T}}} - N_i^{l^{\mathrm{T}}} B_i^{m^{\mathrm{T}}},$$

$$\Pi_3 = 2Q_i(A_i^{l^{\mathrm{T}}} + A_i^{m^{\mathrm{T}}} + A_i^{s^{\mathrm{T}}}) - (N_i^{l^{\mathrm{T}}} + N_i^{m^{\mathrm{T}}}) B_i^{s^{\mathrm{T}}} - (N_i^{m^{\mathrm{T}}} + N_i^{s^{\mathrm{T}}}) B_i^{l^{\mathrm{T}}} - (N_i^{l^{\mathrm{T}}} + N_i^{s^{\mathrm{T}}}) B_i^{m^{\mathrm{T}}}.$$

基于上述定理，给出分散模糊控制设计步骤如下：

第一步 选取隶属函数，构造模型规则 (1)；

第二步 求解线性矩阵不等式 (6)-(9)，得到正定矩阵 Q_i 和 N_i^l，进而得 $K_i^l = N_i^l Q_i^{-1}$；

第三步 设计模糊分散控制器 (4)；

第四步 应用模糊分散控制器 (4) 控制模糊分散系统 (2).

4 数值仿真

考虑如下离散互联模糊系统

$$F: \begin{cases} x_1(t+1) = \sum\limits_{l=1}^{3} \mu_1^l \big[A_1^l x_1(t+1) + B_1^l u_1 + R_{12}^l x_2(t)\big], \\ x_2(t+1) = \sum\limits_{l=1}^{3} \mu_2^l \big[A_2^l x_2(t+1) + B_2^l u_2 + R_{21}^l x_1(t)\big], \end{cases} \tag{10}$$

其中

$$x_1(t)=\left[\begin{array}{cc} x_{11}(t) & x_{12}(t) \end{array}\right]^{\mathrm{T}}, \quad x_2(t)=\left[\begin{array}{cc} x_{21}(t) & x_{22}(t) \end{array}\right]^{\mathrm{T}},$$

$$A_1^1=\left[\begin{array}{cc} 0.05 & 0.15 \\ 0 & 0.42 \end{array}\right], \quad A_1^2=\left[\begin{array}{cc} 0.325 & 0 \\ 0.04 & 0 \end{array}\right], \quad A_1^3=\left[\begin{array}{cc} 0.1 & 0.2 \\ 0.5 & 0 \end{array}\right],$$

$$A_2^1=\left[\begin{array}{cc} 0.4 & 0 \\ 0 & 0.8 \end{array}\right], \quad A_2^2=\left[\begin{array}{cc} 0.4 & 0.5 \\ 0.61 & 0 \end{array}\right], \quad A_2^3=\left[\begin{array}{cc} 0.33 & 0 \\ 0 & 0.41 \end{array}\right],$$

$$R_{12}^1=\left[\begin{array}{cc} 0.1 & 0.1 \\ 0 & 0 \end{array}\right], \quad R_{12}^2=\left[\begin{array}{cc} 0 & 0 \\ 0.1 & 0.1 \end{array}\right], \quad R_{12}^3=\left[\begin{array}{cc} 0.001 & 0.001 \\ 0 & 0 \end{array}\right],$$

$$R_{21}^1=\left[\begin{array}{cc} 0 & 0 \\ 0.1 & 0.2 \end{array}\right], \quad R_{21}^2=\left[\begin{array}{cc} 0 & 0 \\ 0.9436 & 0 \end{array}\right], \quad R_{21}^3=\left[\begin{array}{cc} 0 & 0 \\ 0.9284 & 0 \end{array}\right],$$

$$B_1^1=\left[\begin{array}{c} 1 \\ 0 \end{array}\right], \quad B_1^2=\left[\begin{array}{c} 1 \\ -2 \end{array}\right], \quad B_1^3=\left[\begin{array}{c} 1 \\ 1 \end{array}\right],$$

$$B_2^1=\left[\begin{array}{c} 0 \\ 1 \end{array}\right], \quad B_2^2=\left[\begin{array}{c} -1 \\ 1 \end{array}\right], \quad B_2^3=\left[\begin{array}{c} 1 \\ 1 \end{array}\right].$$

取模糊隶属函数为

$$F_{i1}^l\big(x_{i1}(t)\big)=\frac{1}{1+\exp[-20(x_{i1}(t)-\frac{\pi}{5})]}, \quad F_{i2}^l\big(x_{i1}(t)\big)=\exp\Big(\frac{x_{i1}^2}{0.18}\Big),$$

$$F_{i3}^l\big(x_{i1}(t)\big)=\frac{1}{1+\exp[-20(x_{i1}(t)+\frac{\pi}{5})]}, \quad i=1,2, \quad l=1,2,3,$$

通过求解定理中的线性矩阵不等式(6)-(9)，得到控制增益矩阵

$$\mathrm{K}_1^1=\left[\begin{array}{cc} -0.0863 & 0.0067 \end{array}\right], \quad \mathrm{K}_1^2=\left[\begin{array}{cc} -0.2099 & 0.0688 \end{array}\right], \quad \mathrm{K}_1^3=\left[\begin{array}{cc} -0.0240 & 0.0831 \end{array}\right],$$

$$\mathrm{K}_2^1=\left[\begin{array}{cc} -0.0111 & 0.5800 \end{array}\right], \quad \mathrm{K}_2^2=\left[\begin{array}{cc} -0.5966 & 0.1636 \end{array}\right], \quad \mathrm{K}_2^3=\left[\begin{array}{cc} -0.0736 & 0.2139 \end{array}\right].$$

给定初始值 $x_1(0)=[1,-0.5]^{\mathrm{T}}$，$x_2(0)=[1,0]^{\mathrm{T}}$，得到系统(10)的控制仿真结果．其中图1表示第一个子系统 x_{11} 和 x_{12} 的状态响应曲线，图2表示第二个子系统 x_{21} 和 x_{22} 的状态响应曲线．

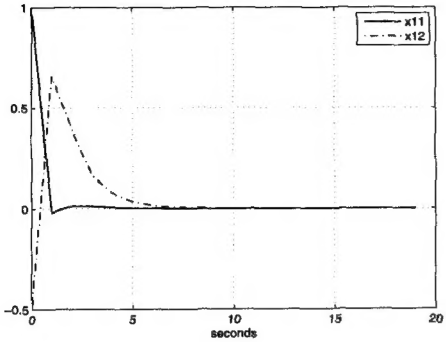

图 1: 子系统 1 的状态响应曲线

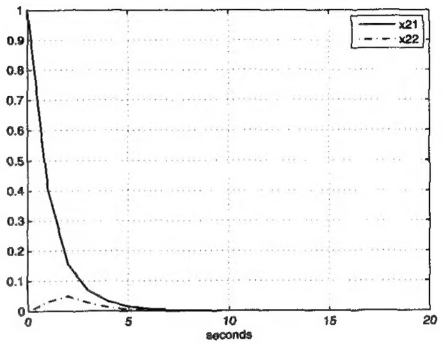

图 2: 子系统 2 的状态响应曲线

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Decentralized Control and Stabilization Criterion for Discrete-time Interconnected Fuzzy Systems

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Abstract: A new decentralized fuzzy control approach is proposed in this paper for a class of discrete-time interconnected fuzzy systems. The sufficient conditions for guaranteeing the stability of the fuzzy control system are given, and the stability of the fuzzy control system is proved by using the Lyapunov function method and linear matrix inequalities theory. The proposed decentralized fuzzy control approach and stabilization conditions can overcome the conservation in the existing fuzzy decentralized methods. The computer simulation results confirm the correction of the propose theory and the effectiveness of the proposed method.

Keywords: interconnected fuzzy systems; decentralized fuzzy control; sufficient conditions; linear matrix inequalities

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